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Quantum controlled-NOT gate with 'hot' trapped ions

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Abstract. We present a novel method of performing quantum logic gates in trapped ion quantum computers which does not require the ions to be cooled down to the ground state of their vibrational modes, thereby avoiding one of the principal experimental difficulties encountered in realizing this technology. Our scheme employs adiabatic passages and a phase shift conditional on the phonon number state.

Not long ago it was recognized by a number of workers that computations exploiting the quantum mechanical features of nature can perform efficiently certain tasks which are intractable with a classical computer [1–3]. This discovery has motivated intensive research into apparatus which could be used to perform quantum logic operations on single or multiple quantum two-level systems ('qubits'). Several possible physical implementations have been suggested, one of them being bulk nuclear magnetic resonance (NMR) [4]. However, there will be very serious problems in implementing large scale computations using high temperature bulk NMR [5], and, as has been very recently demonstrated, preparation of true entangled states with small numbers of qubits in this technology is problematic at best [6]. A scalable low-temperature NMR device has been proposed [7], but there exist many formidable technological problems to be overcome before even simple quantum logic operations can be performed using it. Thus at the moment, ion trap quantum computation, first proposed by Cirac and Zoller [8], and demonstrated experimentally by Monroe *et al.* shortly afterwards [9], is, arguably, the most promising quantum computation technology for realizing systems of dozens of qubits in the foreseeable future. An ion trap quantum computer consists of a string of ions in a linear radio-frequency trap. Two internal states of the ion compose each qubit and the normal modes of the ions' collective oscillations act as a quantum information bus, by means of which quantum logic gate operations can be performed between pairs of ions.

One of the most daunting of the technological problems which must be overcome in the realization of ion trap quantum computation using Cirac and Zoller's scheme is the preparation of multiple ions in the quantum ground state of their collective oscillation modes. Any residual excitation of these modes will

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diminish the accuracy of each logic gate, thus leading to unreliable performance of the quantum computer as a whole, and maybe making the implementation of even simple algorithms impossible. However, reliably pumping multiple ions into their oscillatory ground state, by means of sideband cooling, is a very demanding task which, to date, only one group world-wide has managed to perform [10]. Thus it is very desirable to investigate methods for performing quantum logic gates with 'hot' ions, i.e. ions which have not been cooled down to the quantum mechanical ground state of their oscillatory modes.

Recently, a scheme to avoid the ground-state preparation problem, using ideas adapted from atomic interferometry was proposed [11]. It requires an internal state-dependent momentum kick applied to one ion, of sufficient strength to allow the ion's wavepackets to evolve into two spatially resolvable components. However, for various reasons, this method cannot be used for more than two or three ions, thus making it not applicable for large scale quantum computation. Furthermore the peculiar trapping potentials it proposes (e.g. an axial confining potential $\sim |x|^{5/3}$ instead of the more usual x^2 harmonic confining potential) result in a further complication to experiments which are already very involved and difficult.

Another possibility to avoid the heating problem was pointed out by Sorensen and Mølmer [12]. They make use of 'virtual vibrational excitation' to couple the internal states of two ions to each other. With an appropriate pulse sequence they thus achieve a controlled-NOT gate. Their pulse sequence takes of the order of milliseconds. It is not clear how the so far measured heating rates will affect the gate operation.

In this paper we propose an entirely different approach to the problem of performing quantum logic with 'hot' trapped ions. We make use of the fact that although the ions are not necessarily in the phonon mode ground state, they all share the same phonon mode, thus enabling them to interact with each other. An additional condition on the phonon mode is that, except for the centre-of-mass (CM) mode all the other modes have to be in their motional ground state. This is a drawback, however, the mode which is mainly affected by heating and hard to keep in the motional ground state is the CM vibrational mode. Our scheme performs a controlled-rotation (C-ROT) gate between a pair of ions designated control (c) and target (t). To get a controlled-NOT gate (C-NOT) we only need to sandwich the C-ROT gate between single qubit rotations, which do not depend on the vibrational state and thus are not of concern when heating is considered. The gate operation here consists of a conditional sign change which takes place only if both ions are in the excited state. It can be realized by a sequence of four laser pulses, illustrated symbolically in figure 1.

First a conditional phase shift $\mathcal{S}_t$ is performed between the target qubit and the phonon mode, thereby changing the sign of the wavefunction only if the phonon mode has an odd excitation and the target ion is in its excited state. This operation can be performed, for example, by applying a detuned laser pulse of well-defined duration with the ion at the node of a standing wave of the addressing laser [13] (other experimental arrangements for doing this are discussed below). The next step is to excite an additional phonon into the phonon mode, conditional on the state of the control ion. This is realized by an adiabatic passage between the excited state $|1\rangle_c$ and some auxiliary state $|2\rangle_c$ of the control ion, which at the same time excites a single phonon (see figure 2), thereby changing an even phonon state to an odd phonon state (and vice versa). The advantage of using adiabatic passage for

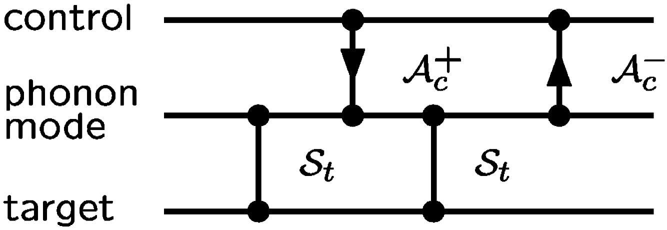

Figure 1. Schematic illustration of the steps involved in the C-ROT gate with hot ions. The individual steps are discussed in detail in the text.

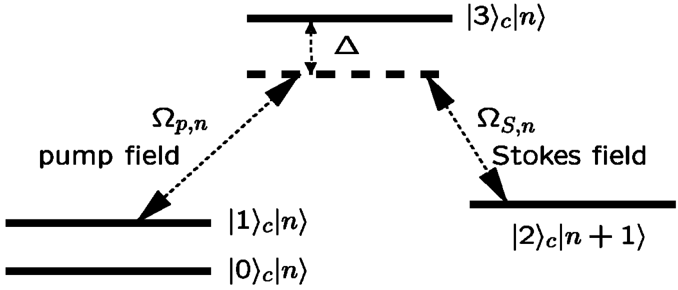

Figure 2. Schematic illustration of the level scheme of the control ion used to realize the adiabatic passage operations $\mathcal{A}_c^+$ and $\mathcal{A}_c^-$.

this step is that the operation can be carried out independent of the number of phonons. The next step is to perform a second conditional sign change $\mathcal{S}_t$. Finally we disentangle the ion states from the phonon mode by performing the adiabatic passage backwards. As will be shown in detail below, these four pulses produce the desired quantum logic gate assuming that, except for the CM mode, all the other modes are in their motional ground state. We will now describe these steps in detail. First let us consider the various laser–ion interactions we will need.

To simplify our analysis we will assume that the phonon mode is in a pure state given by the following formula:

$$|\phi\rangle_{\mathrm{ph}} = \sum_n a_n |n\rangle, \tag{1}$$

where a_n are a set of unknown complex coefficients and $|n\rangle$ is the Fock state of occupation number n. It will be convenient in what follows to introduce the odd and even parts of this wavefunction, namely:

$$\left.\begin{aligned} |\mathrm{even}\rangle_{\mathrm{ph}} &= \sum_n a_{2n}|2n\rangle, \\ |\mathrm{odd}\rangle_{\mathrm{ph}} &= \sum_n a_{2n+1}|2n+1\rangle. \end{aligned}\right\} \tag{2}$$

We will also use the following notation for phonon states to which a single quantum has been added:

$$\left.\begin{aligned} |\mathrm{odd}'\rangle_{\mathrm{ph}} &= \sum_n a_{2n}|2n+1\rangle, \\ |\mathrm{even}'\rangle_{\mathrm{ph}} &= \sum_n a_{2n+1}|2n+2\rangle. \end{aligned}\right\} \tag{3}$$

We should emphasize that our scheme does not require that the CM phonon mode be prepared in either of these states: we are introducing these states for notational convenience only.

The conditional phase change between odd phonon number states and the excited internal state of an ion can be carried out using an effect first considered by D'Helon and Milburn [13]. They introduced a Hamiltonian for a two-level ion at the node of a detuned classical standing wave. In the limit of large detuning and for interaction times much greater than the vibrational period of the trap, this Hamiltonian for the jth ion is

$$H^{(j)} = \hbar \sum_{i=0}^{N-1} a_i^{\dagger} a_i \chi (\sigma_z^{(j)} + 1/2), \tag{4}$$

where $\sigma_z^{(j)}$ is the population inversion operator for the jth ion, a_i and $a_i^{\dagger}$ are the annihilation and creation operators of the ith of the N phonon modes, and $\chi = \eta^2 \Omega^2/(N\delta)$. Here η is the Lamb–Dicke parameter, Ω is the Rabi frequency for the transitions between the two internal states of the ions, N is the total number of ions and δ is the detuning between the laser and the electronic transition. If we choose the duration τ of this interaction to be $\tau = \pi/\chi$, the time evolution is represented by the operator

$$\mathcal{S}_j = \exp\left[-\mathrm{i}\sum_{i=0}^{N-1} a_i^{\dagger} a_i (\sigma_z^{(j)} + 1/2)\pi\right]. \tag{5}$$

If we now assume that the CM mode is in an arbitrary vibrational state, but all the other modes are in their respective motional ground state, then this time evolution flips the phase of the ion when the phonon mode is in an odd state and the ion is in its excited state, thus providing us with a conditional phase shift for an ion and the CM phonon mode.

The adiabatic passage [14] which we require for our gate operation can be realized as follows. We use two lasers, traditionally called the pump and the Stokes field. The pump laser is polarized to couple the qubit state $|1\rangle_c$ to some second auxiliary state $|3\rangle_c$ and is detuned by an amount Δ. The Stokes laser couples to the red side band transition $|2\rangle_c|n+1\rangle \leftrightarrow |3\rangle_c|n\rangle$, with the same detuning Δ. If the population we want to transfer adiabatically is initially in the state $|1\rangle_c|n\rangle$, we turn on the Stokes field (i.e. the sideband laser) and then slowly turn on the pump field (i.e. the carrier laser) until both lasers are turned on fully. Then we slowly turn off the Stokes laser: this is the famous 'counter-intuitive' pulse sequence used in adiabatic passage techniques [14]. The adiabatic passage has to be performed very slowly. The condition in our scheme is that $T \gg 1/\Omega_{\mathrm{p},n}, 1/\Omega_{\mathrm{S},n}$, where T is the duration of the adiabatic passage and $\Omega_{\mathrm{p},n}$ ($\Omega_{\mathrm{S},n}$) are the effective Rabi frequencies for the pump and the Stokes transition, respectively. Furthermore, in order that the various phonon number states be well resolved, we require that $\Omega_{\mathrm{p},n}, \Omega_{\mathrm{S},n} \ll \omega_{\mathrm{ph}}$, the phonon angular frequency. Using the adiabatic passage we can transfer the population from $|1\rangle_c|n\rangle$ to $|2\rangle_c|n+1\rangle$. To invert the adiabatic passage, we just have to interchange the roles of the pump and the Stokes field. We will denote the adiabatic passage by operators $\mathcal{A}_j^+$ and $\mathcal{A}_j^-$ defined as follows:

$$\begin{aligned} \mathcal{A}_j^+ &: |1\rangle_j |n\rangle \rightarrow |2\rangle_j |n+1\rangle, \\ \mathcal{A}_j^- &: |2\rangle_j |n+1\rangle \rightarrow |1\rangle_j |n\rangle. \end{aligned} \tag{6}$$

Putting all these operations together, in detail we can write down the step-by-step states for our gate. We first perform the controlled phase shift between the target ion and the phonon mode. Since this involves distinguishing even and odd phonon mode states, we split them up in our representation, as described above:

$$\begin{aligned} |0\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{S}_t} |0\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\}, \\ |0\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{S}_t} |0\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} - |\text{odd}\rangle_{ph}\}, \\ |1\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{S}_t} |1\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\}, \\ |1\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{S}_t} |1\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} - |\text{odd}\rangle_{ph}\}. \end{aligned} \tag{7}$$

The next step is the adiabatic passage as illustrated in figure 2 and explained above:

$$\begin{aligned} |0\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{A}_c^+} |0\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\}, \\ |0\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} - |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{A}_c^+} |0\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} - |\text{odd}\rangle_{ph}\}, \\ |1\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{A}_c^+} |2\rangle_c|0\rangle_t\{|\text{odd}'\rangle_{ph} + |\text{even}'\rangle_{ph}\}, \\ |1\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} - |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{A}_c^+} |2\rangle_c|1\rangle_t\{|\text{odd}'\rangle_{ph} - |\text{even}'\rangle_{ph}\}. \end{aligned} \tag{8}$$

The next step is the conditional phase flip on the target ion and the CM phonon mode:

$$\begin{aligned} |0\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{S}_t} |0\rangle_c|0\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\}, \\ |0\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} - |\text{odd}\rangle_{ph}\} &\xrightarrow{\mathcal{S}_t} |0\rangle_c|1\rangle_t\{|\text{even}\rangle_{ph} + |\text{odd}\rangle_{ph}\}, \\ |2\rangle_c|0\rangle_t\{|\text{odd}'\rangle_{ph} + |\text{even}'\rangle_{ph}\} &\xrightarrow{\mathcal{S}_t} |2\rangle_c|0\rangle_t\{|\text{odd}'\rangle_{ph} + |\text{even}'\rangle_{ph}\}, \\ |2\rangle_c|1\rangle_t\{|\text{odd}'\rangle_{ph} - |\text{even}'\rangle_{ph}\} &\xrightarrow{\mathcal{S}_t} |2\rangle_c|1\rangle_t\{-|\text{odd}'\rangle_{ph} - |\text{even}'\rangle_{ph}\}. \end{aligned} \tag{9}$$

The last step is the adiabatic passage backwards and the inversion of the rotation on the target ion:

$$
\begin{aligned}
|0\rangle_c|0\rangle_t\Big\{|\mathrm{even}\rangle_{ph} + |\mathrm{odd}\rangle_{ph}\Big\} &\xrightarrow{\mathcal{A}_c^-} |0\rangle_c|0\rangle_t\Big\{|\mathrm{even}\rangle_{ph} + |\mathrm{odd}\rangle_{ph}\Big\},\\
|0\rangle_c|1\rangle_t\Big\{|\mathrm{even}\rangle_{ph} + |\mathrm{odd}\rangle_{ph}\Big\} &\xrightarrow{\mathcal{A}_c^-} |0\rangle_c|1\rangle_t\Big\{|\mathrm{even}\rangle_{ph} + |\mathrm{odd}\rangle_{ph}\Big\},\\
|2\rangle_c|0\rangle_t\Big\{|\mathrm{odd}'\rangle_{ph} + |\mathrm{even}'\rangle_{ph}\Big\} &\xrightarrow{\mathcal{A}_c^-} |1\rangle_c|0\rangle_t\Big\{|\mathrm{even}\rangle_{ph} + |\mathrm{odd}\rangle_{ph}\Big\},\\
|2\rangle_c|1\rangle_t\Big\{-|\mathrm{odd}'\rangle_{ph} - |\mathrm{even}'\rangle_{ph}\Big\} &\xrightarrow{\mathcal{A}_c^-} -|1\rangle_c|1\rangle_t\Big\{|\mathrm{even}\rangle_{ph} + |\mathrm{odd}\rangle_{ph}\Big\}.
\end{aligned}
\tag{10}
$$

Thus we end up with a controlled-rotation gate between the ions c and t. A controlled-NOT (CNOT) gate can be realized by performing $\pi/2$ rotation pulses on the target qubit both before and after this series of operations.

For simplicity, we have analysed these operations under the assumption that the state of the CM phonon mode can be described by an arbitrary pure state. More generally, one must assume that the CM phonon mode is in a mixed state, because it can be entangled with some unknown external quantum system, for example the electromagnetic field causing the heating. Provided we assume that this external system does not become entangled with internal degrees of freedom of the qubits, one can quite easily analyse the gate using a density matrix formalism appropriate for mixed states. Since the adiabatic passage and the conditional phase shift all work for arbitrary CM phonon mode states, our principal result, that gate operations can be performed between arbitrary pairs of qubits, can be shown to be true under these circumstances.

The conditional phase change operation $\mathcal{S}_t$ described above requires the use of standing waves, which are difficult to realize in practice. An alternative method is to use a Raman scheme. Supposing that the two levels $|0\rangle_j$ and $|1\rangle_j$ of the jth ion are the $m_J = -1/2$ and $m_J = 1/2$ sublevels of a ${}^2S_{1/2}$ state, and that the pump and Stokes beams were polarized perpendicular to the quantization axis and to each other, one can show that the effective Hamiltonian of the two levels has the following form:

$$
\begin{aligned}
H^{(j)} &= \chi\sigma_z^{(j)} \cos\left[(\mathbf{k}_p - \mathbf{k}_s).\mathbf{r} - (\omega_p - \omega_s)t\right],\\
&\approx \chi\sigma_z^{(j)}\left[1 - \eta^2\left(a^\dagger a + \frac{1}{2}\right) + O\left(\eta^4\right)\right],
\end{aligned}
\tag{11}
$$

where η is the Lamb–Dicke parameter, χ is a constant, and we have assumed that $\omega_p = \omega_s$ and that terms in $\exp(\pm i2\omega_{ph}t)$ can be neglected. Although this is not quite the same form as the Hamiltonian given by equation (4), the difference results in a phase shift which can be corrected by a further single-ion laser pulse.

An important source of error in performing gate operations using this scheme is the heating during gate operations. To perform logic operations, effectively the quantum information stored in the two levels of the control qubit is transferred to the even and odd states of the phonon mode. Heating mixes these two states, thereby degrading the information stored. As mentioned above, the adiabatic passage must be performed over a time period T which is much greater than the inverse of the phonon frequency (which is usually in the range $(2\pi)100$ kHz to $(2\pi)10$ MHz in most experiments). Thus the duration of gate operations will be of the order of microseconds in this scheme. This is definitely a drawback of our scheme, given that the current technology heating times are roughly of this order [10], however, in the future it might be possible to use our scheme to perform quantum logic experiments without the necessity of careful preparation of the quantum state of the CM mode.

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